Local magnetic moments of spin $1/2$ in a hole doped antiferromagnetic spin chain

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We investigate local magnetic moments of spin $1/2$ in a hole doped antiferromagnetic spin chain. In the hole doped spin chain massive spinon excitations of spin $1/2$ are found owing to the contribution of the doped holes [R. Shankar, Phys. Rev. Lett. **63**, 203 (1989)], which is in contrast with the undoped spin chain where massless spinons are obtained. Including the Kondo coupling between the massive spinons and the local spins and integrating over the massive spinons, we obtain an effective Hamiltonian for the local spins. The Kondo coupling is shown to result in local interactions between the local spins. This contribution leads the local spins to behave as the Tomonaga-Luttinger liquid in the half-filled case of the local spins. Further, even in the case of the isotropic Heisenberg coupling between the local spins, the Umklapp scattering does not cause usual logarithmic corrections in correlation functions of the local spins owing to the contribution of the massive spinons.

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Recently we showed that in a hole doped antiferromagnetic spin chain a magnetic moment of spin $1/2$ emerges in a non-magnetic impurity[1]. The local magnetic moment results from a spinon of spin $1/2$ trapped in the non-magnetic impurity[1]. In the present study we investigate the localized spins in the hole doped antiferromagnetic spin chain [Eq. (1)]. Here we consider the half filled case of the localized spins [Eq. (1)]. Then this problem becomes an extended version of the Kondo problem. In the Kondo problem scatterings between local magnetic moments and non-interacting conduction electrons are taken into account. On the other hand, the present study considers local magnetic moments of spin $1/2$ interacting with strongly correlated electrons.

In the absence of interactions between the conduction electrons the problem is usually described by the Kondo lattice model[2]. It seems to be clearly established that in the case of the half-filled conduction band the ground state is the Kondo spin liquid (Kondo insulator) for any non-zero Kondo coupling in one spacial dimension, which has a gap to both spin and charge excitations[3, 4, 5]. The behavior away from the half filling (hole doping to the conduction band) remains controversial[3, 4]. Paramagnetic metallic phase may be expected[3, 4, 6]. Numerical studies available until now seems to support the Tomonaga-Luttinger liquid (TLL) with dominant correlations determined by the conduction electrons[3, 4, 6].

Here we consider strongly correlated electrons, the doped antiferromagnetic spin chain instead of the non-interacting conduction electrons. The doped spin chain is described by spinons and holons interacting via U(1) gauge fields [Eq. (2)]. The spinons carry spin $1/2$ of electrons and the holons, charge $+e$ of doped holes. The Kondo coupling between the spinons and the local spins is taken into account [Eq. (5)]. The main message of the present study is as follows. The presence of the holons leads the spinons to be massive[1, 7] [Eq. (4)]. Excitations of the massive spinons result in local interactions between the local spins via the Kondo coupling [Eq. (7)]. This contribution causes the expectation value of the local moments to vanish. As a result a paramagnetic phase is expected to occur. We find that the local spins behave as the TLL [Eq. (16)]. Further, it is found that the Umklapp scattering does not cause usual logarithmic corrections in correlation functions of the local spins owing to the contribution of the massive spinons [Eq. (18)].

We consider a doped antiferromagnetic spin chain in the presence of local magnetic moments

$$
\begin{aligned}
H &= H_{t-J} + H_{imp} + H_K, \\
H_{t-J} &= -t\sum_{i=1}^{N}(c_{\sigma i}^{\dagger}c_{\sigma i+1} + h.c.) + J\sum_{i=1}^{N}\mathbf{s}_i\cdot\mathbf{s}_{i+1}, \\
H_{imp} &= I\sum_{i=1}^{N}\Big(\tau_i^x\cdot\tau_{i+1}^x + \tau_i^y\cdot\tau_{i+1}^y + \Delta\tau_i^z\cdot\tau_{i+1}^z\Big), \\
H_K &= J_K\sum_{i=1}^{N}\mathbf{s}_i\cdot\tau_i.
\end{aligned} \tag{1}
$$

Here the $t-J$ Hamiltonian H_{t-J} describes the doped antiferromagnetic spin chain and the Hamiltonian H_{imp}, the local moments. Δ denotes Ising anisotropy. The correlated electrons and the local moments are antiferromagnetically correlated via the Kondo coupling H_K. If strong correlations represented by the Heisenberg coupling $J\sum_{i=1}^{N}\mathbf{s}_i\cdot\mathbf{s}_{i+1}$ and the no-double-occupancy constraint $\sum_{\sigma=1}^{2}c_{\sigma i}^{\dagger}c_{\sigma i}\leq 1$ are neglected, this Hamiltonian is reduced to the Kondo lattice model[2]. Here we deal with both the strong correlations and the local moments on an equal footing in the presence of hole doping. As far as we know, no one has treated both effects on an equal footing in the presence of hole doping.

For the time being, we focus on the $t-J$ Hamiltonian. In the absence of hole doping hopping of electrons is suppressed and thus the $t-J$ Hamiltonian is reduced to the Heisenberg Hamiltonian describing a quantum antiferromagnetic spin chain. Low energy physics of the quantum spin chain can be described by a non-linear σ model with a Berry phase term[8]. Utilizing CP^1 representation, one can represent the non-linear σ model in terms of bosonic spinons interacting via compact U(1) gauge fields in the presence of the Berry phase[8]. Considering hole doping to the spin chain, Shankar showed that the doped holes are represented by massless Dirac fermions dubbed holons and the fermionic holons interact with the bosonic spinons via the U(1) gauge fields[7]. A low energy effective field theory is obtained to be[7]

$$S_{t-J} = \int d^2x \Big[|(\partial_\mu - i a_\mu) z_\sigma|^2 + m^2 |z_\sigma|^2 + \frac{u}{2}(|z_\sigma|^2)^2 - iS\epsilon_{\mu\nu}\partial_\mu a_\nu \Big] + \int d^2x \Big[\bar{\psi}_A \gamma_\mu (\partial_\mu + i a_\mu)\psi_A + \bar{\psi}_B \gamma_\mu (\partial_\mu - i a_\mu)\psi_B \Big] \quad (2)$$

Here z_σ is a bosonic spinon (spin) and $\psi_{A(B)}$, a fermionic holon (charge) in a sublattice $A(B)$. The spinons and holons interact via the compact U(1) gauge field a_μ. m is a mass of the spinon and u, a strength of a local interaction[9]. S in the Berry phase term $iS\epsilon_{\mu\nu}\partial_\mu a_\nu$ represents the value of spin 1/2. A detailed derivation is given by Ref. [7]. The presence of the massless Dirac fermions completely alters the situation in the absence of those[7, 10]. First, the massless Dirac fermions are shown to kill the Berry phase effect[10]. As a result the spinons are expected to be massive[7, 10, 11]. Second, the contribution of the massless Dirac fermions results in the massive gauge field. Thus gauge fluctuations are suppressed in the low energy limit and the spinons and holons are expected to be deconfined[7, 10]. Last, superconducting correlations between the charge degree of freedom increase[7]. In order to see this physics one can utilize the standard bosonization method[7, 10]

$$\bar{\psi}_A \gamma_\mu \partial_\mu \psi_A = \frac{1}{2}(\partial_\mu \phi_A)^2, \;\; \bar{\psi}_B \gamma_\mu \partial_\mu \psi_B = \frac{1}{2}(\partial_\mu \phi_B)^2, \quad \bar{\psi}_A \gamma_\mu \psi_A = \frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial_\nu \phi_A, \;\; \bar{\psi}_B \gamma_\mu \psi_B = \frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial_\nu \phi_B \quad (3)$$

Here ϕ_A and ϕ_B are bosonic fields in each sublattice. Inserting these into the above action Eq. (2), one obtains an effective action[7, 10]

$$S_{t-J} = \int d^2x \Big[|(\partial_\mu - i a_\mu) z_\sigma|^2 + m^2 |z_\sigma|^2 + \frac{u}{2}(|z_\sigma|^2)^2 \Big] + \int d^2x \Big[\frac{1}{2}(\partial_\mu \phi_+)^2 + \frac{1}{2}(\partial_\mu \phi_-)^2 + i\sqrt{\frac{2}{\pi}}\phi_- \epsilon_{\mu\nu}\partial_\mu a_\nu - iS\epsilon_{\mu\nu}\partial_\mu a_\nu \Big] \quad (4)$$

with $\phi_+ = \frac{1}{\sqrt{2}}(\phi_A + \phi_B)$ and $\phi_- = \frac{1}{\sqrt{2}}(\phi_A - \phi_B)$. Shifting the ϕ_- field to $\phi_- + \sqrt{\frac{\pi}{2}}S$, one can easily see that the Berry phase term is wiped out from the action. Thus the half-odd integer spin chain is not distinguishable from the integer spin chain[11]. The bosonic spinon in the doped half-odd integer spin chain is expected to be massive like that in the undoped integer spin chain[7, 10, 11]. But the spinons here are not confined in contrast with the case of the integer spin chain[7, 10]. Integrating over the ϕ_- field, we find that the U(1) gauge field becomes massive and thus it is ignorable in the low energy limit. As a consequence the massive spinons are deconfined[7, 10]. A spin liquid with massive spinon excitations emerges in the doped antiferromagnetic spin chain. If we introduce an electromagnetic field A_μ, we obtain a coupling term of $i\sqrt{\frac{2}{\pi}}\phi_+\epsilon_{\mu\nu}\partial_\mu A_\nu$. Integrating over the ϕ_+ field, we obtain a mass of the electromagnetic field. This implies superconductivity in the doped spin chain, which is consistent with the result of Shankar[7]. In summary, the doped holes lead the spin degree of freedom to be the gapped spin liquid and the charge degree of freedom to be superconducting. It should be noted that this result is exact in the low energy limit[7].

Integrating over all bosonic fields ϕ_+, ϕ_-, and a_μ except the spinons z_σ and introducing the Kondo coupling between the spinons and the local spins, we obtain an effective action

$$S = S_{t-J} + S_K + S_{imp}, \quad S_{t-J} + S_K = \int d^2x \Big[|\partial_\mu z_\sigma|^2 + m^2|z_\sigma|^2 + \frac{J_K}{2} z_\alpha^\dagger \sigma^k_{\alpha\beta} z_\beta \tau^k \Big], \quad (5)$$

where S_{imp} is associated with the impurity Hamiltonian H_{imp} in Eq. (1) in the presence of the Berry phase contribution for the local spins. In the above a local interaction between the spinon currents originating from integration over the massive gauge field a_μ is not explicitly taken into account because it is expected to be irrelevant (marginally) in the low energy limit. We also omit the local interaction between the spinons. Since it is usually introduced for phase transitions, it is not necessary any more. In the previous study[1] we examined the role of non-magnetic impurities in the hole doped one dimensional Mott insulator. As discussed above, in the doped Mott insulator the spinons are massive. As a result the spinon is localized in the non-magnetic impurity and the impurity behaves as a magnetic one[1]. Here we are considering the local magnetic moments in the doped Mott insulator.

At first glance, it seems to be trivial to solve Eq. (5) since the spinons are massive. The mass is a relevant parameter in the renormalization group sense. The lower the energy scale, the larger the mass. As a result it is expected that the massive spinons do not affect the localized spins. But the Kondo coupling is expected to be also a relevant parameter. It also becomes larger as the energy scale gets lower. In this respect the spinons are expected to affect the local moments. The competition between the mass of the spinons and the Kondo coupling is expected. Since the two relevant parameters are taken into account, the problem is not so simple.

Integrating over the massive spinons and expanding a resulting logarithmic term to the second order in the

Kondo coupling J_K, we obtain

$$\tilde{\mathcal{L}} = -Trln\Big[-\partial^2 + m^2 + \frac{J_K}{2}\sigma\cdot\tau\Big]$$
$$\approx -Trln\Big[-\partial^2 + m^2\Big]$$
$$+Tr\Big(\frac{1}{2}\frac{J_K^2}{4}(\sigma\cdot\tau)_x\Pi(x-x')(\sigma\cdot\tau)_{x'}\Big)$$
$$= -Trln\Big[-\partial^2 + m^2\Big] + \frac{1}{2}\frac{J_K^2}{4}\tau_x\Pi(x-x')\tau_{x'},$$
$$\Pi(q) = \int\frac{d^2k}{(2\pi)^2}\frac{1}{(k+q)^2+m^2}\frac{1}{k^2+m^2}. \quad (6)$$

Here $\Pi(q)$ is the polarization function of the massive spinons in momentum space. In real space it exponentially decays owing to the mass of the spinons[12]. A local interaction between the localized spins is expected to occur owing to the Kondo coupling with the massive spinons. We propose a model of the local interaction. In the Hamiltonian formalism we obtain an effective Hamiltonian in terms of the local spins

$$H_{eff} = H_{imp} + \tilde{H},$$
$$H_{imp} = I\sum_{i=1}^{N}\Big(\tau_i^x\cdot\tau_{i+1}^x + \tau_i^y\cdot\tau_{i+1}^y + \Delta\tau_i^z\cdot\tau_{i+1}^z\Big),$$
$$\tilde{H} = \lambda\sum_{i=1}^{N}\tau_i\cdot\tau_i$$
$$= \lambda\sum_{i=1}^{N}\Big(\tau_i^x\cdot\tau_i^x + \tau_i^y\cdot\tau_i^y + \tau_i^z\cdot\tau_i^z\Big) \quad (7)$$

with the coupling constant λ. λ is a function of J_K and m^2. The Kondo coupling between the massive spinons and the local moments results in the local interaction between the local spins, which is modelled by $\tilde{H}$. This contribution is expected to cause the expectation value of the local spins to vanish. If the local spins have a finite expectation value, the Kondo coupling is expected to result in Kondo singlets between the massive spinons and the local spins. Since the spinons are massive, it costs much energy to form the Kondo singlets. As a result it is energetically more favorable that the expectation value of the local spins vanishes. We check this argument using the Abelian bosonization.

First we introduce the Jordan-Wigner (JW) transformation[8]

$$\tau_i^+ = f_i^\dagger K(i) = K(i)f_i^\dagger,$$
$$\tau_i^- = f_iK(i) = K(i)f_i,$$
$$K(i) = e^{i\pi\sum_{j=1}^{i-1}f_j^\dagger f_j} = e^{i\pi\sum_{j=1}^{i-1}(\tau_i^z+\frac{1}{2})} \quad (8)$$

with the spin raising and lowering operators, $\tau^+ = \tau^x + i\tau^y$ and $\tau^- = \tau^x - i\tau^y$, respectively. Here f_i is a JW fermion which represents a kink configuration with a non-local operator $K(i)$[8]. Inserting this fermion into the above Hamiltonian [Eq. (7)] and performing some algebra[13], we obtain an effective Hamiltonian in terms of the JW fermions

$$H_{eff} = H_{xy} + H_z + \tilde{H},$$
$$H_{xy} = \frac{I}{2}\sum_{i=1}^{N}(f_i^\dagger f_{i+1} + f_{i+1}f_i^\dagger),$$
$$H_z = I\Delta\sum_{i=1}^{N}(f_i^\dagger f_i - \frac{1}{2})(f_{i+1}^\dagger f_{i+1} - \frac{1}{2}),$$
$$\tilde{H} = \lambda\sum_{i=1}^{N}(f_i^\dagger f_i + \frac{1}{4}). \quad (9)$$

H_{xy} is the contribution of the XY spin components and H_z, that of the Z spin component. As well known, the Ising coupling term is expressed as an interaction between the JW fermions[8]. $\tilde{H}$ is considered as a chemical potential term of the JW fermions. Excitations of the massive spinons control the density of the kink excitations.

Next we represent the JW fermion in terms of a right handed fermion R_i and a left handed one L_i, i.e., $f_i = R_ie^{ik_Fx_i} + L_ie^{-ik_Fx_i}$ with the Fermi momentum k_F. Using $f_i^\dagger f_i - \frac{1}{2} =: f_i^\dagger f_i := [: R_i^\dagger R_i : + : L_i^\dagger L_i :] + [e^{-2ik_Fx_i} : R_i^\dagger L_i : + e^{2ik_Fx_i} : L_i^\dagger R_i :]$ where $: \hat{O} :$ is the normal ordering of an operator $\hat{O}$[8], we obtain a quadratic Hamiltonian

$$H_{xy} = \int dr\Big[\pi I(\rho_R(x)^2 + \rho_L(x)^2)\Big],$$
$$H_z = \int dr\Big[I\Delta(\rho_R(x)+\rho_L(x))^2 - I\Delta(\chi(x)+\chi^\dagger(x))^2\Big],$$
$$\tilde{H} = \int dr\Big[\lambda(\rho_R(x)+\rho_L(x)+\chi(x)+\chi^\dagger(x))\Big], \quad (10)$$

where $\rho_R(x), \rho_L(x)$ and $\chi(x), \chi^\dagger(x)$ are given by

$$\rho_R(x) =: R(x)^\dagger R(x) :,$$
$$\rho_L(x) =: L(x)^\dagger L(x) :,$$
$$\chi(x) = e^{-2ik_Fx} : R^\dagger(x)L(x) :,$$
$$\chi^\dagger(x) = e^{2ik_Fx} : L^\dagger(x)R(x) :, \quad (11)$$

respectively. ρ_R and ρ_L represent the density of the right and left handed fermions respectively. χ and χ^+ are associated with scattering between the right and left handed fermions. Now utilizing the standard bosonization of each fermion $R(x)$ and $L(x)$

$$R(x) = \frac{1}{\sqrt{2\pi\alpha}}\eta_1e^{i\phi_R(x)} = \frac{1}{\sqrt{2\pi\alpha}}\eta_1e^{i\frac{\theta_+(x)+\theta_-(x)}{2}},$$
$$L(x) = \frac{1}{\sqrt{2\pi\alpha}}\eta_2e^{-i\phi_L(x)} = \frac{1}{\sqrt{2\pi\alpha}}\eta_2e^{i\frac{-\theta_+(x)+\theta_-(x)}{2}} \quad (12)$$

with the Klein factors η_1 and η_2[8], we can represent ρ

and χ in terms of the bosonic fields θ_+ and θ_-

$$\rho_R(x) = \frac{1}{2\pi}\partial_r\phi_R(x) = \frac{1}{4\pi}(\partial_r\theta_+(x) + \partial_r\theta_-(x)),$$
$$\rho_L(x) = \frac{1}{2\pi}\partial_r\phi_L(x) = \frac{1}{4\pi}(\partial_r\theta_+(x) - \partial_r\theta_-(x)),$$
$$\chi(x) = \frac{1}{2\pi\alpha}\eta_1\eta_2 e^{-i\theta_+(x)-2ik_F r},$$
$$\chi^\dagger(x) = \frac{1}{2\pi\alpha}\eta_2\eta_1 e^{i\theta_+(x)+2ik_F r}. \quad (13)$$

Here the parameter α is a cut-off with magnitude $\alpha \sim 1$[8]. Inserting these into Eq. (10), we obtain an effective Hamiltonian in terms of the bosonic fields

$$H_{eff} = H_{imp} + \tilde{H},$$
$$H_{imp} = \int dr\Big[\Big(\frac{I}{8\pi} + \frac{I\Delta}{4\pi^2}\Big)(\partial_r\theta_+)^2 + \frac{I}{8\pi}(\partial_r\theta_-)^2 - \frac{I\Delta}{2(\pi\alpha)^2}\cos 2\theta_+\Big]$$
$$\tilde{H} = \int dr\Big[\frac{\lambda}{2\pi}\partial_r\theta_+ + \frac{\lambda}{\pi\alpha}\eta_2\eta_1\sin(\theta_+ + 2k_F r)\Big]. \quad (14)$$

H_{imp} is the same as a usual bosonized Hamiltonian of the Heisenberg spin chain with the Ising anisotropy[8]. As well known, the cos term in H_{imp} results from the Umklapp scattering between the left and right handed fermions[8]. $\tilde{H}$ is the new one. $\partial_r\theta_+$ is associated with the total density of the right and left handed fermions, i.e., $\rho \equiv \rho_R + \rho_L = \frac{1}{2\pi}\partial_r\theta_+$. The emergence of the total density term seems to be natural owing to the chemical potential term of the JW fermion resulting from the Kondo coupling. The $\sin(\theta_+ + 2k_F r)$ term seems to be due to the fact that the massive spinon excitations scatter the right handed fermion to the left handed fermion. Below we argue that the cos and sin terms are irrelevant owing to the linear derivative term, $\partial_r\theta_+$ in $\tilde{H}$.

Shifting θ_+ to $\tilde{\theta}_+ - \frac{\lambda}{4\pi}\Big(\frac{I}{8\pi} + \frac{I\Delta}{4\pi^2}\Big)^{-1} r$, the first term of $\tilde{H}$ in Eq. (14) is wiped out. As a result we obtain an effective Hamiltonian

$$H_{eff} = \int dr\Big[\Big(\frac{I}{8\pi} + \frac{I\Delta}{4\pi^2}\Big)(\partial_r\tilde{\theta}_+)^2 + \frac{I}{8\pi}(\partial_r\theta_-)^2 - \frac{I\Delta}{2(\pi\alpha)^2}\cos\Big(2\tilde{\theta}_+ - \frac{\lambda}{2\pi}\Big(\frac{I}{8\pi} + \frac{I\Delta}{4\pi^2}\Big)^{-1} r\Big) + \frac{\lambda}{\pi\alpha}\eta_2\eta_1\sin\Big(\tilde{\theta}_+ + 2\Big[k_F - \frac{\lambda}{8\pi}\Big(\frac{I}{8\pi} + \frac{I\Delta}{4\pi^2}\Big)^{-1}\Big]r\Big)\Big]. \quad (15)$$

In the absence of the Kondo coupling, i.e., $\lambda = 0$, this effective Hamiltonian describes the Heisenberg spin chain with the Ising anisotropy[8]. In the easy plane limit of $\Delta \ll 1$ the Umklapp scattering represented by the cos term is irrelevant and thus the Tomonaga-Luttinger liquid (TLL) is obtained[8]. In the easy axis limit of $\Delta \gg 1$ the cos term is relevant and the antiferromagnetic long range order is expected[8]. In the isotropic case of $\Delta = 1$ the Umklapp scattering is marginally irrelevant[14] and thus the TLL is still expected. But in this case the Umklapp scattering results in logarithmic corrections to correlation functions[14, 15]. On the other hand, in the present case the Umklapp scattering is expected to be irrelevant owing to the contribution of the massive spinons. Excitations of the massive spinons cause the chemical potential term to the kink excitations. This leads to the linear derivative term of the bosonic field θ_+. Shifting the θ_+ field to $\tilde{\theta}_+$, we obtain the cos term oscillating in space. Owing to the spacial oscillation it is expected to be irrelevant[16]. The sin term is also expected be irrelevant because of the same reason. As a consequence the local spins are expected to behave as the TLL independent of the Ising anisotropy

$$H_{eff} = \int dr\Big[\Big(\frac{I}{8\pi} + \frac{I\Delta}{4\pi^2}\Big)(\partial_r\tilde{\theta}_+)^2 + \frac{I}{8\pi}(\partial_r\theta_-)^2\Big]. \quad (16)$$

Physically this result can be understood as follows. If the Umklapp scattering becomes relevant and thus it causes antiferromagnetic ordering of the local spins, the spinons are expected to form the Kondo singlets owing to the Kondo coupling. But, since the spinons are massive, it is energetically unfavorable to form the Kondo singlets. Thus the paramagnetic state is naturally expected. However, because the Kondo coupling is a relevant parameter, it affects the local moments. As a result the boson field θ_+ is renormalized to $\tilde{\theta}_+$. In terms of the renormalized boson field $\tilde{\theta}_+$ the TLL is obtained.

Now we discuss correlation functions. In the absence of the massive bosonic spinons z_σ the isotropic Heisenberg coupling between the local spins leads the spin-spin correlation function of staggered components to be at zero temperature[15]

$$< \tau^z(r,\tau)\tau^z(0,0) > \sim \frac{(-1)^r}{(r^2+\tau^2)^{1/2}} ln^{\frac{1}{2}}\sqrt{r^2+\tau^2} \quad (17)$$

with the position r and the imaginary time τ. The logarithmic correction results from the Umklapp scattering as pointed out earlier[15]. Similarly, the staggered susceptibility has a logarithmic correction factor $\chi_s(T) \sim T^{-1}ln^{1/2}T^{-1}$ owing to the same reason. However, in the present case the isotropic limit of $\Delta = 1$ does not cause the logarithmic correction. This is because the Umklapp scattering becomes irrelevant owing to the contribution of the massive spinons [Eq. (15) and Eq. (16)]. As a result the spin-spin correlation function of the staggered components is given by at zero temperature[14, 17]

$$< \tau^z(r,\tau)\tau^z(0,0) > \sim \frac{(-1)^r}{(r^2+\tau^2)^{1/2}}. \quad (18)$$

Temperature dependence of specific heat $C(T)$ and staggered susceptibility $\chi_s(T)$ is given by $C(T) \sim T$ and $\chi_s(T) \sim T^{-1}$, respectively[14].

To summarize, we investigated the effect of strongly correlated electrons on local magnetic moments in the one dimensional hole doped Mott insulator in the weak

Kondo coupling regime. Strong correlation effect causes the doped Mott insulator to be the gapped spin liquid for the spin degree of freedom. We found that half filled local magnetic moments of spin 1/2 in the one dimensional gapped spin liquid behave as the Tomonaga-Luttinger liquid owing to the contribution of the massive spinons via the Kondo coupling. Further, even in the case of the isotropic Heisenberg coupling between the local spins, the Umklapp scattering becomes irrelevant owing to the contribution of the massive spinons and thus no logarithmic corrections in the spin-spin correlation function, specific heat, and staggered susceptibility are found.

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[11] In the case of integer spin the Berry phase is ignorable. Strong quantum fluctuations originating from low dimensionality lead the bosonic spinons to be massive[8]. The integer spin chain becomes disordered. The massive spinons are confined via a linearly increasing gauge potential in a distance which results from Maxwell kinetic energy of the gauge field[9]. As a consequence mesons (spinon-antispinon bound states, here spin excitons) are expected to appear[9]. In the case of half-odd integer spin the Berry phase plays a crucial role to cause destructive interference between quantum fluctuations, thus weakening quantum fluctuations. Owing to the Berry phase contribution the half-odd integer spin chain is expected to be ordered. But low dimensionality leads the system not to be ordered but to be critical[8]. As a result the spinons are massless[9]. The massless spinons are expected to be deconfined[9] because critical fluctuations of the spinons can weaken gauge fluctuations via screening.
[12] Using the Feynman identity, we express the polarization function $\Pi(q)$ as

$$\Pi(q) = \int \frac{d^2k}{(2\pi)^2}\frac{1}{(k+q)^2+m^2}\frac{1}{k^2+m^2}$$
$$= \int_0^1 dz \int \frac{d^2k}{(2\pi)^2}\frac{1}{\Big((1-z)(k^2+m^2)+z[(k+q)^2+m^2]\Big)^2}.$$

A change of variables $k \to k - zq$ turns $\Pi(q)$ into

$$\Pi(q) = \int_0^1 dz \int \frac{d^2k}{(2\pi)^2}\frac{1}{\Big(k^2+z(1-z)q^2+m^2\Big)^2}.$$

Utilizing Mathematica 5.0, we can perform the momentum k integral and obtain

$$\Pi(q) = \frac{1}{4\pi}\int_0^1 dz\frac{1}{m^2+z(1-z)q^2}.$$

Now we perform the Fourie transformation of $\Pi(q)$ to get $\Pi(x)$. Using Mathematica 5.0, we obtain

$$\Pi(x) = \int \frac{d^2q}{(2\pi)^2}e^{iqx}\Pi(q)$$
$$= \frac{1}{(2\pi)^2}\int dqd\theta q e^{iq|x|\cos\theta}\Pi(q) = \frac{1}{2\pi}\int_0^\infty dqqJ_0(q|x|)\Pi(q)$$
$$= \frac{1}{8\pi^2}\int_0^1 dz\int_0^\infty dq\frac{qJ_0(qx)}{m^2+z(1-z)q^2}$$
$$= \frac{1}{8\pi^2}\int_0^1 dz\frac{K_0\Big(\sqrt{\frac{m^2x^2}{z(1-z)}}\Big)}{z(1-z)}.$$

Here $J_0(x)$ is the zeroth order Bessel function of the first kind. $K_0(x)$ is associated with the zeroth order Hankel function. Changing the integration variable z into y via the relation of $mx/\sqrt{z(1-z)} = y$, we obtain

$$\Pi(x) = \frac{1}{2\pi^2}\int_{2mx}^\infty dy\frac{K_0(y)}{\sqrt{y^2-4m^2x^2}}.$$

In the limit of $y \gg 2mx$, using the asymptotic form of $K_0(y) = \sqrt{\pi/2}(e^{-y}/\sqrt{y})$ [G. B. Arfken and H. J. Weber, Mathematical Methods For Physicists, Fourth Edition (Ch. 11), Academic Press (1995)], we obtain the asymptotic form of the polarization function

$$\Pi(x) \approx \frac{1}{(2\pi)^{\frac{3}{2}}}\int_{2mx}^\infty dy\Big[\frac{e^{-y}}{y^{\frac{3}{2}}}+2m^2x^2\frac{e^{-y}}{y^{\frac{7}{2}}}\Big] \sim \frac{e^{-2mx}}{\sqrt{mx}}.$$

This integration is also performed by Mathematica 5.0.
[13] The self-interaction τ_i^2 is represented in terms of the JW fermion as

$$\frac{1}{2}(\tau_i^+\tau_i^- + \tau_i^-\tau_i^+) + \tau_i^z\tau_i^z = f_i^\dagger f_i + (f_i^\dagger f_i - \frac{1}{2})^2$$
$$= f_i^\dagger f_i + f_i^\dagger f_i f_i^\dagger f_i - f_i^\dagger f_i + \frac{1}{4}$$
$$= f_i^\dagger f_i + f_i^\dagger f_i - f_i^\dagger f_i + \frac{1}{4} = f_i^\dagger f_i + \frac{1}{4}.$$

Here we used the relation of $f_i^\dagger f_i f_i^\dagger f_i = f_i^\dagger f_i$.
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